Retrieval of Optimal Subspace Clusters Set for an Effective Similarity Search in a High Dimensional Spaces

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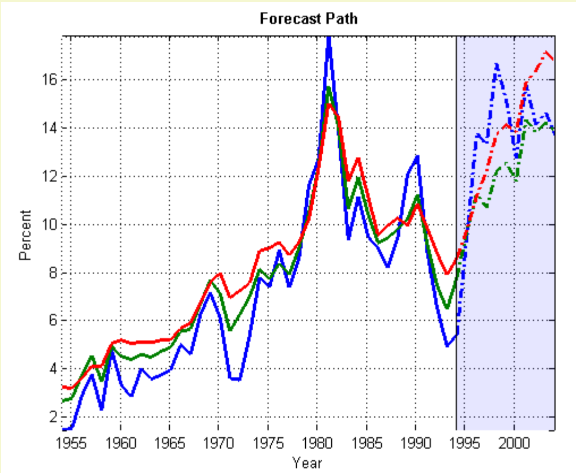
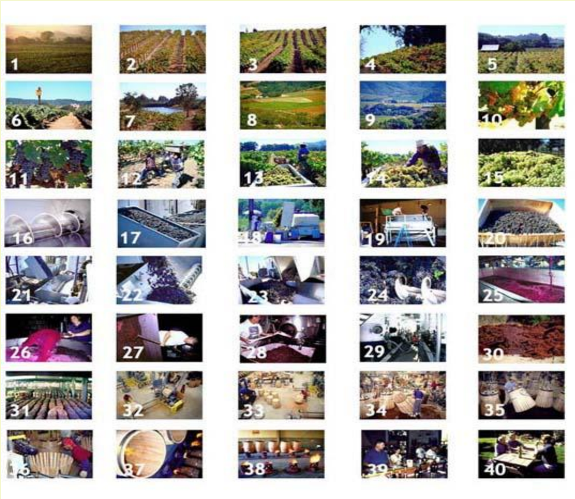
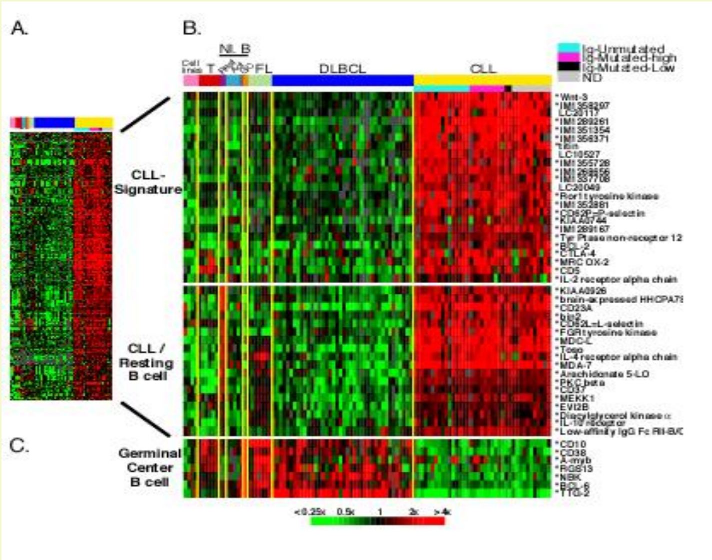
Retrieval problem



Time series



Media collections



Gene expressions

(0,4,10,15,0,-1,1,3,16,10,1,-2)
(0,2,-1,-5,0,-1,2,6,11,-1,4,2)
(0,4,-5,-3,0,-2,1,1,2,7,19,21)
(0,1,6,4,9,-4,-2,-5,-3,7,15,5)

Vector space; d >= 10

Retrieval problem

(0,4,10,15,0,-1,1,3,16,10,1,-2)
(0,2,-1,-5,0,-1,2,6,11,-1,4,2)
(0,4,-5,-3,0,-2,1,1,2,7,19,21)
(0,1,6,4,9,-4,-2,-5,-3,7,15,5)
....
(0,2,-1,-5,1,1,2,6,10,-1,4,3)
....
(1,1,-3,-5,1,1,2,7,10,-1,4,3)
....
(2,0,-1,-4,3,6,2,7,10,-1,4,3)

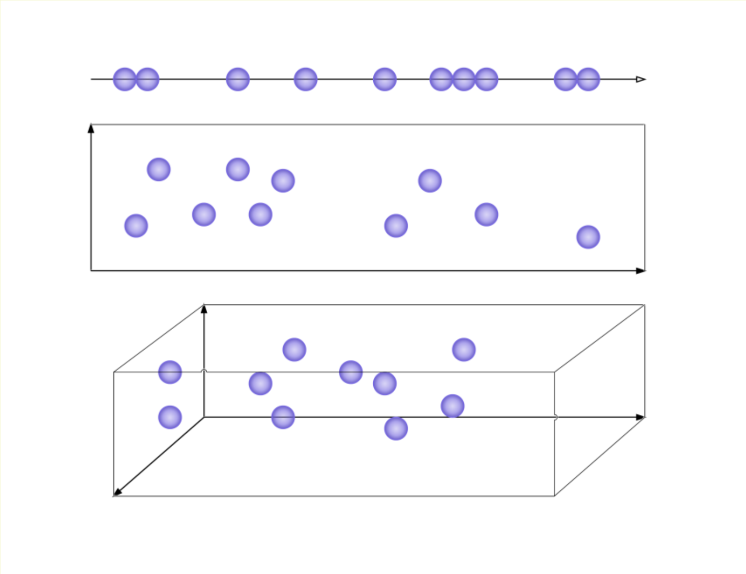
q = (0,2,-1,-5,0,-1,3,7,12,0,2,2)

Curse of dimensionality

- Near equidistant in terms od Euclid distance

(0,4,10,15,0,-1,1,3,16,10,1,-2)
(0,2,-1,-5,0,-1,2,6,11,-1,4,2)
(0,4,-5,-3,0,-2,1,1,2,7,19,21)
(0,1,6,4,9,-4,-2,-5,-3,7,15,5)

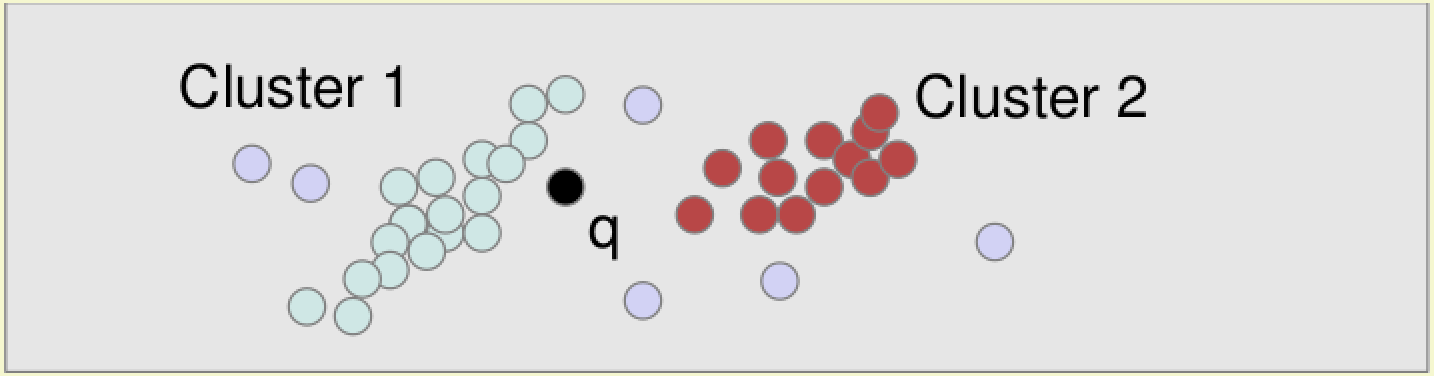
- Space volume grows exponentially
- Number of attributes under analysis is high



Inapplicability of low-dimensional indexing techniques

- Space partitioning index structures (like R-trees) lead to exponentially explosion of index size.
- Random projection leads to inability of relevant nearest neighbour detection

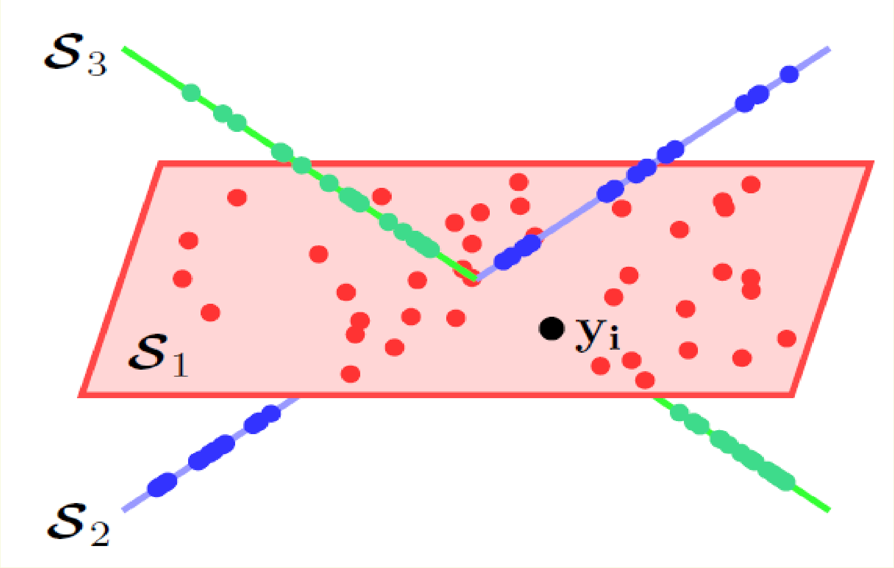
Clustering approach to indexing



- Clustering is a convinient form of unsupervised learning;
- Cluster has a role of vector-approximation file
- Retrieval task is reduced to classification

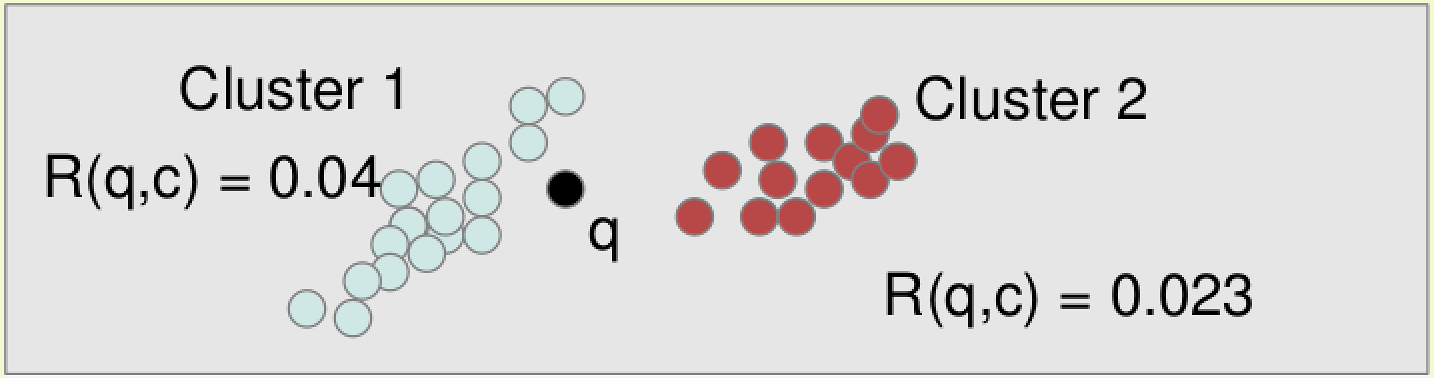
Subspace and projection clustering

- An approach to fight curse of dimensionality for clustering
- Particular data points form clusters in a particular subspace



Subspace Cluster = {S,C};
where S — subset of dimensions;
C- subset of data vectors

Retrieval approach with subspace clusters



$$R(c,q) = R_{dim}(c,q)\frac{\sum_{v \in V} \frac{1}{dist(q,v)}}{k}$$

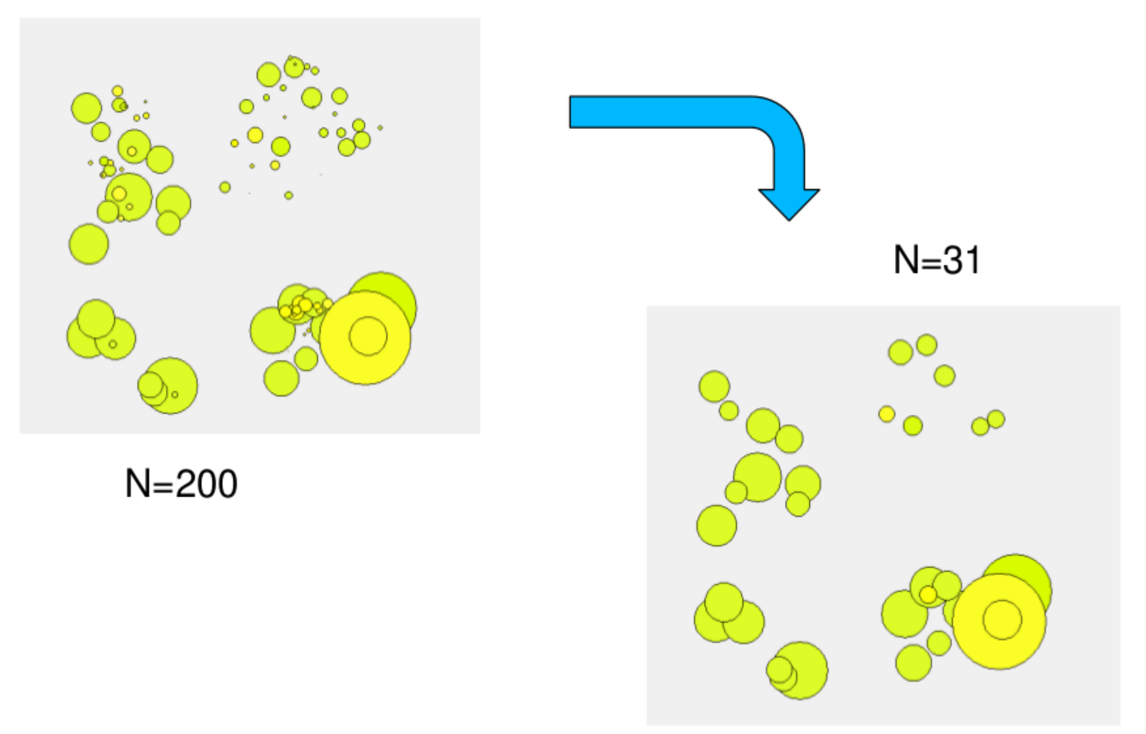
$$R_{aprox}(c,q) = R_{dim}(c,q)Q(c,q)$$

- Calculate an approximate relevance of the cluster with respect to a given query
- Select the set of the most relevant clusters
- Continue refinements

Problem: curse of dimensionality again

- **The number of all clusters in all possible subspaces can be significantly high**
- Cluster in subspace of dimensionality > 5 is still complex to analyse; Q becomes complex;

Problem: what we can do



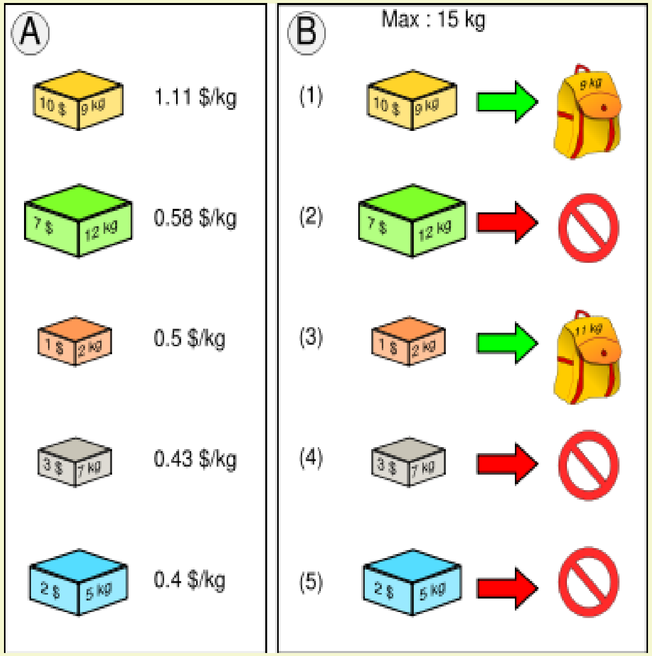
Consider only the best (optimal) subset of clusters as an index set

Optimization problem

$$
\begin{cases}
E(R(c^*, q)) \to max, \\
\quad where \quad c^* = \arg\max_{c \in C} R_{aprox}(c, q) \\
N \leq N_{max} \\
E(g_q(|V|, |s|)) \leq \gamma \forall q
\end{cases}
$$

Solution

- Hard to solve in a general way; Say, q distributed uniformely, then we need to calculate complex multiple integral sums;
- We will attempt to reduce the problem to a knapsack problem
- We need weights and values;



Solution: Knapsack problem reduction

- Weight of the cluster can be assigned to 1
  w = 1
- To introduce value function an estimation algorithm was invented;

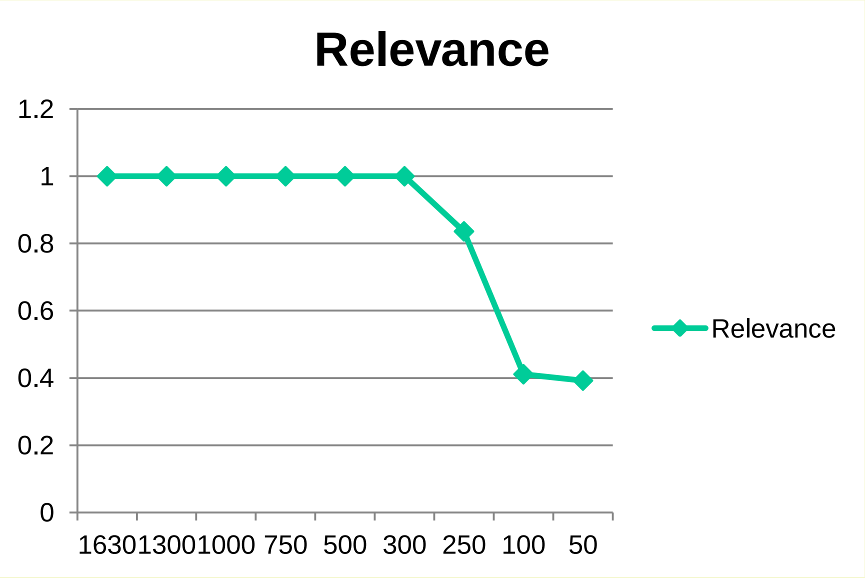
$$(u(x), w(x)) := (R(x, c_i); R_{approx}(x, c_i))$$

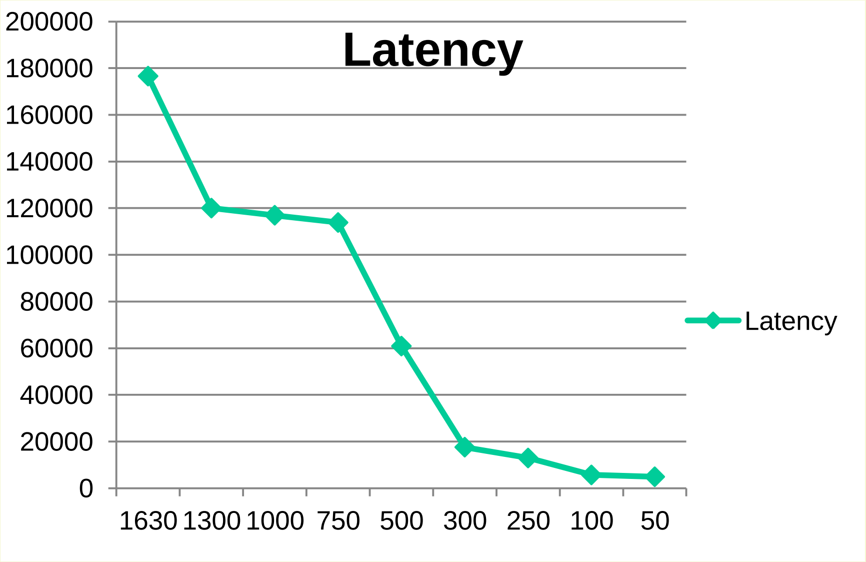
$$v_i = corr(u, w)$$

Solution: Results

- Weather observation data
- ~17 millions of vectors
- Dimensionality 200
- 1630 clusters detected by MAFIA
- Average dimensionality 9.5

Solution: Results





Q & A